A STUDY OF SPONTANEOUS CONDENSATION OVER A WIDE RANGE OF INITIAL PRESSURES

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An experimental and theoretical study has been made concerning the flow of saturated vapor through a Laval nozzle with attendant spontaneous condensation, over the range of initial pressures from 2.35 to 29.40 bars. This study has shown that the initial pressure has an appreciable effect on the spontaneous condensation process.

In this study the authors used as the working medium saturated vapor entering the test device under various pressures. The test device was a Laval nozzle with axial symmetry and the following design parameters: the profile of the 0.040 m long entrance segment was shaped by joining the arcs of two circles with a radius 0.033 m and 0.012 m respectively, the diameter of the inlet section was 0.066 m, the diameter of the throat section was 0.008 m, and the 0.027 m long diverging segment was conical with a 6° vertex angle. The design value of the Mach number, based on $k = 1.3$, was $Ma = 2$.

The operating part of the test apparatus shown in Fig. 1 consisted of a receiver 1, the test nozzle 2, and an exhaust segment 3. Both the receiver and the nozzle were made of stainless steel.

In order to study the flow of water vapor, we made the following sets of measurements involving the parameters of the medium:

1. The static pressure at the nozzle entrance p_0, at the nozzle throat p_m, and in the exhaust line (back pressure) p_e.

2. The distribution of static pressure along the nozzle.

For this purpose we used an axial probe 4 (Fig. 1), one end of which was fastened to a coordinate table 5 and the other end was coupled to the load 6 through a cable. The probe position along the nozzle axis was each time fixed by means of guides 7.

The location of the probe for sampling the static pressure was checked by means of a rule with 0.1 mm divisions, along the coordinate table. The initial reading was throughout the experiment corrected

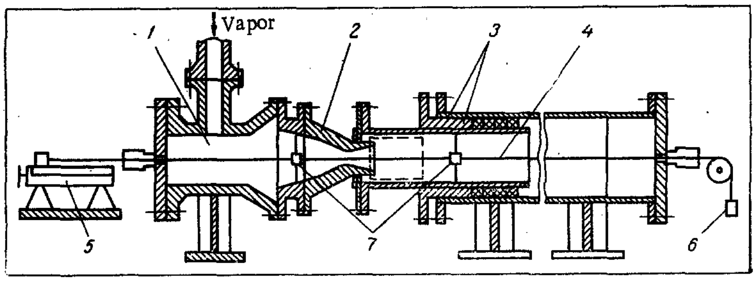

Fig. 1. Schematic diagram of the test apparatus: 1) receiver, 2) nozzle, 3) exhaust segment, 4) axial probe, 5) coordinate table, 6) load, 7) guides.

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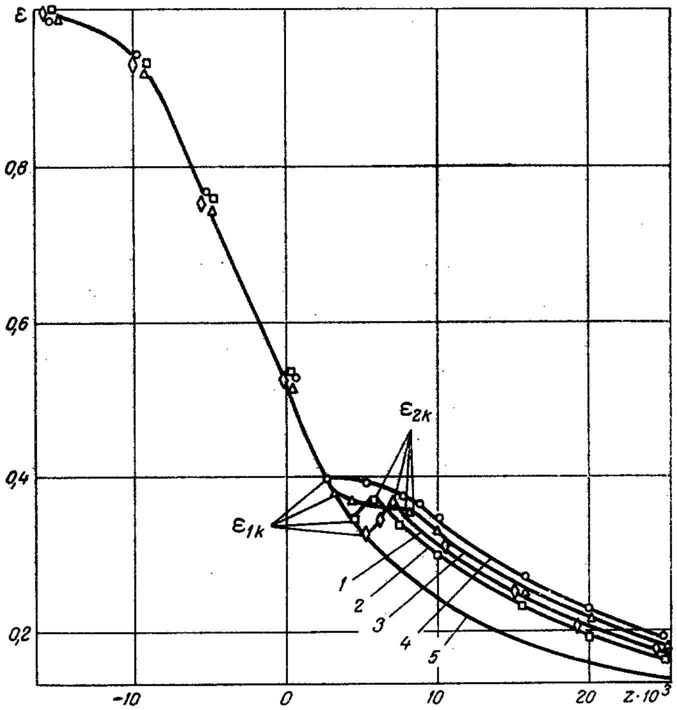

Fig. 2. Distribution of static pressure along the nozzle (m): $p_0 = 2.35$ bars (1), 6.26 bars (2), 16.40 bars (3), 29.40 bars (4), curve 5 plotted for $k = 1.3$ (ε_{1k} and ε_{2k} beginning and end, respectively, of the zone where the effect of phase transformations is strong).

periodically by two methods: visually, by the position of the sampling hole at the nozzle throat section (sighting clearances were provided, for this purpose, in the exhaust pipe with optical glass windows), and by comparing the probe readings with the readings at the reference sampling point near the nozzle throat.

The experiment has shown the pressure to be almost the same at the nozzle axis and at the nozzle walls (except for the zone of spontaneous condensation).

3. The initial vapor temperature, measured with a Chromel—Copel thermocouple.

The tests were performed at a vapor pressure before the nozzle 2.35, 6.28, 16.40, and 29.40 bars and at initial temperatures equal to the respective saturation temperatures.

The results of this experimental study are shown in Figs. 2 and 3.

The distribution of static pressure along the nozzle are shown in Fig. 2 for $p_0 = 2.35$, 6.26, 16.40, and 29.40 bars. It is evident from these curves that the trend of the pressure curves changes as p_0 increases: at $p_0 = 2.35$ and 6.26 bars, there is a positive pressure gradient within the zone where the effect of phase transformations is strong ($\varepsilon_{1k} - \varepsilon_{2k}$), while at $p_0 = 16.40$ and 29.40 bars the pressure gradient is negative along the entire nozzle (also within the $\varepsilon_{1k} - \varepsilon_{2k}$ zone). As p_0 increases, ε_{1k} shifts upstream toward the throat section.

The maximum subcooling, the fictitious relative dryness (determined from the i—s diagram for equilibrium vapor flow up to pressure p_{1k}), and the supersaturation are shown in Fig. 3 as functions of the initial pressure. As p_0 is increased, ΔT_{max} decreases from 55°C (at $p_0 = 2.35$ bars) to 50°C (at $p_0 = 29.40$ bars) and x_{1k} decreases respectively from 0.943 to 0.926. The relative dryness $p_{1k}/(p_{1k})_s$ changes most abruptly from 11.7 to 3.5, with $(p_{1k})_s$ denoting the saturation pressure at temperature T_{1k} (which is determined from gasodynamic tables).

Thus, the results of experimental studies indicate that the initial pressure has an appreciable effect on the process of spontaneous condensation.

In order to interpret the test data, calculations based on the kinetics of phase transformations were made with the aid of a digital computer.

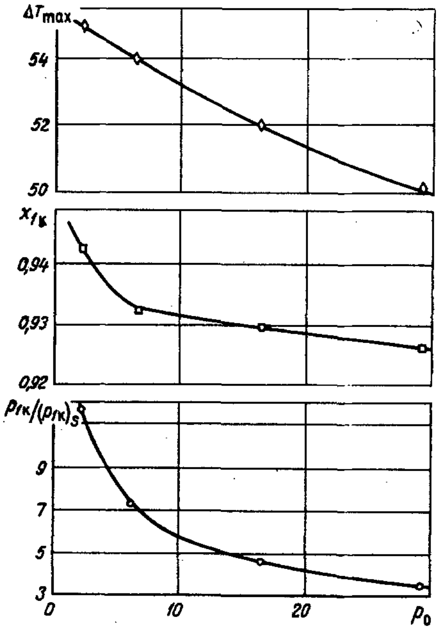

Fig. 3. Maximum subcooling (°C), fictitious relative dryness, and supersaturation, as functions of the initial pressure.

As the basis for these calculations we used the differential equations of motion for the vapor phase given in [1], assuming:

1. a steady and one-dimensional flow;
2. equal velocities of both phases, $c_1 = c_2 = c$;
3. no effect of viscous and volume forces;
4. a negligibly small volume of the condensate phase, as compared to the total volume.

With these assumptions, the system of differential equations includes: the continuity equation

$$\frac{d}{dz}(\rho_1 cF) = -\varkappa F; \tag{1}$$

the momentum equation

$$\rho_1 c \frac{dc}{dz} = -\frac{dp}{dz}; \tag{2}$$

the equation of stagnation enthalpy

$$\rho_1 c \frac{di_{01}}{dz} = \varkappa(i_1 - \bar{\mu}_3) + Q. \tag{3}$$

In order to integrate a system of differential equations numerically, one must reduce it to the form

$$\frac{dy_i}{dz} = f(z;\ y_1;\ y_2;\ \ldots,\ y_n), \quad i = 1,\ 2,\ \ldots,\ n. \tag{4}$$

This system of equations was, furthermore, supplemented with:

1. the equation of thermal state in the form

$$p_1 = \bar{z}RT_1\rho_1$$

with the compressibility factor $\bar{z}$, accounting for the deviation of the vapor from an ideal gas, assumed here constant for each mode and equal to its mean value over any given pressure range;

2. the equation of thermal state in terms of enthalpy

$$i_1 = \frac{k}{k-1}\cdot\frac{p_1}{\rho_1} + \text{const};$$

3. the equation for the rate of phase transformations

$$\varkappa = \frac{1}{F(z)}\int_{z_0}^{z} J(\xi)\,\frac{dm(\xi,\ z)}{dz}\,F(\xi)\,d\xi + J(z)\,m_*(z),$$

$$z_0 \leqslant \xi \leqslant z.$$

So far several formulas have been derived for the rate of vapor nucleation J, different quantitatively but similar in structure to the Frenkel'–Zel'dovich formula [2]. The latter, namely

$$J = \left(\frac{p_1}{kT_1}\right)^2 v_2 \sqrt{\frac{2\sigma}{\mu_1\pi}}\exp\left[\left(-\frac{\Delta W}{kT_1}\right)\beta\right] \tag{5}$$

was used as a basis for our study.

The value of factor β in the exponential term of this formula was determined on the basis of a best agreement between calculated and measured pressure distributions.

The basic role of factor β is that of a correction factor to the work of nucleation calculated according to the formula [3]:

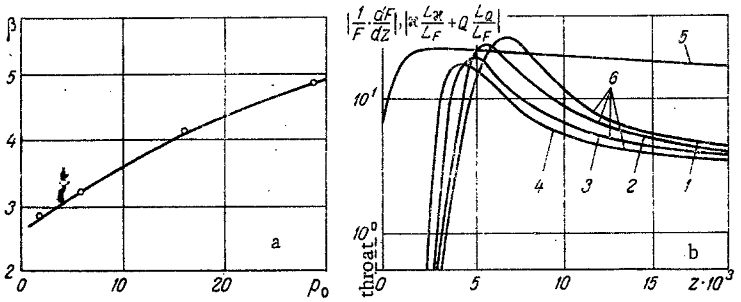

Fig. 4. Some calculated results: (a) coefficient β as a function of the initial pressure, (b) magnitudes of the effects due to phase transformations, due to heat transfer, and due to channel geometry along the nozzle at p_0 = 2.35 bars (1), 6.83 bars (2), 16.40 bars (3), 29.40 bars (4), magnitudes of the effect due to channel geometry (5), total effect due to phase transformations and heat transfer (6). $L_F = a^2c^2p_1$, $L_x = c[a^2-(k-1)T_1S_1]$, $L_Q = -(k-1)c$, pressure p_0 (bars), distance along the nozzle axis z (m).

$$\Delta W = \frac{4}{3}\pi r_{cr}^2\sigma,$$

where the radius of a nucleus has been defined in [3] as $r_{cr} = 2\sigma/\rho_2\Delta\bar{\mu}$ under the following assumptions: first of all, that the coefficient of surface tension, the density of the liquid phase, and the difference of the thermodynamic specific potentials do not depend on the nucleus size and are equal to those respectively defined for aggregates of many molecules; secondly, that the difference of thermodynamic specific potentials is also equal to $\Delta\bar{\mu} = \bar{r}(\Delta T/T_s)$, i.e., the first nonzero term in the expansion of $\Delta\bar{\mu}$ in powers of subcooling ΔT [3].

Calculations have shown that changes in the coefficient β, without affecting the shape of the pressure curve within the zone where phase transformations strongly influence the stream, do affect the relative position of this curve: a higher value of β shifts the curve in the downstream direction. Calculations as well as a comparison with test data have revealed that coefficient β increases monotonically with increasing initial pressure p_0 (Fig. 4a).

The rate of vapor condensation on liquid droplets was determined according to the Knudsen formula [4]:

$$\frac{dm}{dt} = \frac{4\pi r^2 p_1\alpha}{\sqrt{2\pi RT_1}}\left(1-\frac{p_2}{p_1}\sqrt{\frac{T_1}{T_2}}\right) \tag{6}$$

where $p_2 = p_1 + 2\sigma/r$.

For lack of data, as in the case of factor β, the value of the condensation coefficient α was determined on the basis of a comparison between calculated and measured pressure distributions. According to the results of analysis, variations in the condensation coefficient from 1.0 to 0.001 have no appreciable effect on the location of the zone where phase transformations strongly influence the stream, but they very much distort the pressure distribution curve within this zone, which has made it possible to match the values of α and β independently. The most appropriate value seems to be $\alpha = 1$.

The temperature of particles was assumed equal to the saturation temperature.

4. Another supplementary equation was that of heat transfer between vapor and condensate particles

$$Q = \int_{z_0}^{z}\bar{q}(\xi, z)\frac{J(\xi)}{c(\xi)}d\xi \tag{7}$$

with $\bar{q}(\xi, z)$ denoting the quantity of heat received or released at section z by one particle which had nucleated at section ξ. Since ours was a case of free-molecular flow, hence $\bar{q}(\xi, z)$ was determined according to the equation [5]:

$$\bar{q}(\xi,\ z) = 2{,}34\,\pi\rho_1 \sqrt{\frac{2RT_1}{\pi}}\, R\alpha_{\text{T}} \frac{k}{k-1} r^2(\xi,\ z)\,(T_2 - T_1).$$

The coefficient of thermal accommodation α_T was assumed here equal to unity.

According to calculations, the effect of heat transfer was in our case very small.

The trend of curves representing the distribution of static pressure can be interpreted from the one-dimensional viewpoint, if one analyzes the equation for the derivative dp/dz after system (1)-(3) has been reduced to (4):

$$\frac{dp}{dz} = \frac{c^2\rho_1 \left\{ -\varkappa c \left[\frac{i_1 - \bar{\mu}_3}{kp_1} \rho_1 (k-1) - 1 \right] + \frac{1}{F} \cdot \frac{dF}{dz} - Q \frac{k-1}{c} M_1^2 \right\}}{1 - M_1^2}. \tag{8}$$

Equation (8) indicates that a distortion of curve p(z) is due to the effects of phase transformations, due to heat transfer, and due to channel geometry.

Calculated curves describing the strengths of these effects are shown in Fig. 4b. As p_0 is increased, according to these curves, the effects due to phase transformations and due to heat transfer diminish, which causes the p(z) curve to change its shape within the zone where phase transformations have a strong influence, while at $p_0 \gtrsim 16.40$ bars they become even weaker than the effects due to channel geometry. As a result, the total effect on the stream is such that the pressure gradient becomes negative.

A smoother trend of the condensation process with increasing initial pressure p_0 was effected by a decreasing maximum subcooling attainable in the system and, consequently, lower nucleation rates, i.e., a smaller number of condensate nuclei.

Calculations have shown ΔT_{max} to change from 54.4°C at $p_0 = 2.35$ bars to 47.3°C at $p_0 = 29.40$ bars, and the nucleation rate changing respectively from $1.6 \cdot 10^{24}$ to $0.36 \cdot 10^{23}$ $1/m^3 \cdot$ sec.

NOTATION

p	is the static pressure;
T	is the temperature;
a	is the velocity of sound;
z	is the longitudinal coordinate in the nozzle, m;
c	is the velocity;
ρ	is the density;
F	is the area;
x_{ik}	is the fictitious relative dryness;
i_0	is the stagnation enthalpy;
i	is the static enthalpy;
$\varkappa$	is the rate of phase transformations;
$\bar{\mu}_3$	is the thermodynamic specific potential of the phase undergoing aggregate transformation;
Q	is the quantity of heat transmitted by particles convectively;
$\bar{z}$	is the compressibility factor;
R	is the gas constant;
$k = c_p/c_v$;	
c_p	is the specific heat at constant pressure;
c_v	is the specific heat at constant volume;
S	is the entropy;
J	is the nucleation rate;
$m_*(z)$	is the mass of droplets nucleated at section z;
m	is the mass of droplets;
k	is the Boltzmann constant;
v	is the specific volume;
σ	is the coefficient of surface tension;
μ	is the molecular weight;
ΔW	is the work of liquid nucleation;
β	is a coefficient;
$\varepsilon = p/p_0$;	
r_{cr}	is the critical radius of nucleus;

$\Delta\bar{\mu}$	is the difference of thermodynamic specific potentials;
$\bar{r}$	is the specific heat of phase transformations;
α	is the condensation coefficient;
α_T	is the coefficient of thermal accommodation.

Subscripts

1	refers to vapor;
2	refers to liquid;
s	refers to saturation;
0	refers to initial condition.

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